

## NEUTRINO PHYSICS

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This report concerns itself with proposed neutrino and antineutrino experiments for the 200-BeV machine. For the moment, we limit our discussion to two-body reactions and propose to investigate the muon neutrino and muon antineutrino reactions:

$$\nu + n \rightarrow p + \mu^-, \quad \text{"elastic scattering"} \quad (1)$$

$$\nu + p \rightarrow N^{*++} + \mu^-, \quad (2)$$

$$\nu + n \rightarrow N^{*+} + \mu^-, \quad (3)$$

$$\bar{\nu} + p \rightarrow n + \mu^+, \quad \text{"elastic"} \quad (4)$$

$$\bar{\nu} + n \rightarrow N^{*-} + \mu^+, \quad (5)$$

$$\bar{\nu} + p \rightarrow N^{*0} + \mu^+, \quad (6)$$

$$\bar{\nu} + p \rightarrow \Lambda^0 + \mu^+, \quad (7)$$

$$\bar{\nu} + p \rightarrow \Sigma^0 + \mu^+, \quad (8)$$

$$\bar{\nu} + n \rightarrow \Sigma^- + \mu^+, \quad (9)$$

$$\bar{\nu} + n \rightarrow Y_1^{*-} + \mu^+, \quad (10)$$

$$\bar{\nu} + p \rightarrow Y_1^{*0} + \mu^+. \quad (11)$$

Reactions (1), (4), (7)-(9) are examples of unitary octet-octet transitions, whereas (2), (3), (5), (6), (10), and (11) correspond to octet-decuplet transitions. The  $\Delta S = \Delta Q$  reactions (8) and (9), and (10) and (11) allow us to test the selection rule  $\Delta I = 1/2$ , with the

predicted cross-section ratios for (9) to (8), as well as (10) to (11), being 2:1. The  $\Delta S = 0$  reactions (2) and (3), as well as (5) and (6), furnish us with a test of the selection law  $\Delta I = 1$ , with the predicted ratios of (5) to (6) and (2) to (3) being 3:1.

We now turn our attention to a detailed analysis of the "elastic" scattering reactions (1) and (4). We assume, following a suggestion by Cabibbo, that we can write the strangeness nonchanging hadronic current as

$$J^\mu = G \cos \theta \left[ F_V \gamma^\mu + i \mu \frac{F_M}{m} \sigma^{\mu\nu} q_\nu + i a F_S q^\mu - i \lambda F_A \gamma^\mu \gamma_5 + i c \frac{F_E}{m} \sigma^{\mu\nu} q_\nu \gamma_5 + i b F_P q^\mu \gamma_5 \right]. \quad (12)$$

We have allowed for the 6 form factors possible by Lorentz invariance, and these four factors, which are function of  $q^2$  (four-momentum transfer squared) are all normalized to 1 at  $q^2 = 0$ . The phase factors selected are such to make all F's relatively real. G is the universal Fermi constant, and  $\theta$  is the Cabibbo angle. The scale of the weak magnetism has been fixed by the conserved vector current (CVC) hypothesis, with  $\mu$  ( $= 3.71$ ) being the difference in the anomalous magnetic moments of the proton and neutron, with m the nucleon mass. The axial scale is chosen by  $\lambda = -G_A/G_V$  (and thus is positive, and equal to 1.18). The phases in  $J^\mu$  have been chosen so that the normal first-class currents, given by the vector form factor  $F_V$ , the weak magnetism  $F_M$ , the axial  $F_A$  and the induced pseudoscalar  $F_P$ , preserve

time-reversal invariance, whereas the phases for the second-class currents (opposite behavior under the operation CP) have been picked to give a maximal violation of time reversal invariance. The constant  $b$  can be fixed using PCAC, whereas the dimensionless coupling constants for the second-class currents,  $\underline{c}$  and  $\underline{a}$ , are left as unknowns to be fixed by experiment. For the elastic reaction  $\nu + n \rightarrow p + \mu^-$ , we compute, using the current (12), that the spin averaged differential cross section per unit squared momentum transfer is given by

$$\begin{aligned} \frac{d\sigma}{dq^2} = & \frac{G^2}{2\pi} \left\{ F_V^2 + \lambda^2 F_A^2 + \frac{q^2}{4m^2} \left[ (\mu F_M)^2 + (2c F_E)^2 + \right. \right. \\ & + 2 \left( 2\lambda F_A (\mu F_M + F_V) - \lambda^2 F_A^2 - F_V^2 \right) \frac{m}{E_\nu} - \\ & - \left( \lambda F_A (m_\mu b F_p) \left( \frac{m_\mu}{m} \right) - \left( \frac{m_\mu a F_s}{4} \right)^2 - \left( \lambda^2 F_A^2 - F_V^2 \right) \right] \frac{q^2}{E_\nu^2} \Bigg] \\ & + \frac{q^4}{8m^4} \left[ - \left( (\mu F_M)^2 + (2c F_E)^2 \right) \frac{m}{E_\nu} + \frac{m^2}{E_\nu^2} \left( (\lambda F_A - F_V)^2 \right. \right. \\ & \left. \left. - 2\mu F_M (\lambda F_A - F_V) + \left( \frac{\mu F_M}{2} \right)^2 + \left( \frac{m_\mu b F_p}{2} \right)^2 + \left( \frac{m_\mu a F_s}{2} \right)^2 \right) \right] \Bigg\}. \quad (13) \end{aligned}$$

The cross section (13) has been computed in the limit in which we set the muon mass  $m_\mu$  equal to zero, only retaining  $m_\mu$  when otherwise the term would completely vanish, such as for the induced pseudoscalar and scalar terms. We have chosen to express the cross section as an explicit power series in the dimensionless parameters  $q^2/m^2$  and  $m/E_\nu$ .

where  $E_\nu$  is the laboratory neutrino energy. From (13), we note that since all  $F$ 's are normalized to 1 at  $q^2 = 0$ , we get the simple result, valid for all energies  $E_\nu$ , that at  $q^2 = 0$ ,

$$\frac{d\sigma}{dq^2} (q^2 = 0) = \frac{G^2}{2\pi} (1 + \lambda^2) = 1.91 \times 10^{-38} \text{ cm}^2 / (\text{BeV}/c)^2. \quad (14)$$

Since (14) is valid for all  $E_\nu$ , this expression can be used to calibrate internally the neutrino flux as follows. In a bubble chamber we can measure, in various  $E_\nu$  bins, the number of elastic scatters per unit  $q^2$ , at  $q^2 = 0$ , and by virtue of the known cross section (14), we can deduce the neutrino spectrum.

We note that for large enough energy  $E_\nu$ , at a given  $q^2$ , the terms  $m/E_\nu$  appearing in (13) can be neglected, and the resultant differential cross section simplifies to

$$\frac{d\sigma}{dq^2} (q^2, E_\nu \rightarrow \infty) = \frac{G^2}{2\pi} \left\{ F_V^2 + \lambda^2 F_A^2 + \frac{q^2}{4m^2} \left[ (\mu F_M)^2 + (2c F_E)^2 \right] \right\}. \quad (15)$$

Under the assumption that all form factors have the same  $q^2$  dependence, we estimate for  $m/E_\nu \lesssim 0.25$ , that (13) can be replaced by (15) with negligible error. For  $E_\nu \gtrsim 4 \text{ BeV}$ , we see from (15) that  $d\sigma/dq^2$  is independent of energy. Thus, a very simple test of the locality hypothesis is to demonstrate experimentally that  $d\sigma/dq^2$  is independent of energy for neutrino energies  $\gtrsim 4 \text{ BeV}$ .

If we wish to calculate the elastic scattering of antineutrinos,

i. e. (4)  $\bar{\nu} + p \rightarrow n + \mu^+$ , we can easily get it from (12) under the substitution  $+\lambda \rightarrow -\lambda$ . We see from (12) that the first important term, that is different for antineutrino scattering contains  $(q^2/4m^2)(m/E_\nu)$  factors, and thus, to lowest order for small  $(q^2/4m^2)(m/E_\nu)$ ,

$$\frac{d\sigma}{dq^2}(\bar{\nu}) = \frac{G^2}{2\pi} \left\{ F_V^2 + \lambda^2 F_A^2 + \frac{q^2}{4m^2} \left[ (\mu F_M)^2 + (2c F_E)^2 + 2(-2\lambda F_A(\mu F_M + F_V) - \lambda^2 F_A^2 - F_V^2) \frac{m}{E_\nu} \right] + \dots \right\}. \quad (16)$$

Numerical evaluation of (16), compared to (15), shows that, under the assumption that all four factors are the same, the experimental difference between  $\nu$  and  $\bar{\nu}$  scattering at the same momentum transfer, for  $m/E_\nu < 0.5$ , is very small--less than a few percent. For this reason, we stress the requirement: experiments to measure the vector-axial interference terms, i. e., the difference between neutrino and antineutrino elastic scattering, must be carried out in beams with energies in the region 0.5 to 1.5 BeV, in order to have measurably large enough effects. Thus, special neutrino and antineutrino beams, optimized for low-energy flux, are required.

We further note that in the high-energy limit (15), we can conveniently evaluate the total elastic scattering cross section analytically, if we assume that all form factors have the shape of a double pole, i. e.,

$$F_x = \frac{1}{(1 + q^2/m_x^2)^2},$$

where  $m_x$  is the mass associated with the  $x^{\text{th}}$  form factor. Using the

CVC hypothesis, we obtain

$$\sigma_{\text{total}}(E_{\nu} \rightarrow \infty) = \frac{G^2}{2\pi} \left[ \frac{m_V^2}{3} + \frac{\lambda^2 m_A^2}{3} + \left( \frac{\mu m_V}{24} \right)^2 \left( \frac{m_V}{m} \right)^2 + \left( \frac{2cm_E}{24} \right)^2 \left( \frac{m_E}{m} \right)^2 \right]. \quad (17)$$

Letting  $m_V = 0.84$  BeV (from the electron-scattering experiments), and  $m_A = 0.84$  BeV (indicated by the CERN bubble-chamber neutrino results) and setting  $c = 0$  (no weak electricity), we obtain from (17) that

$$\sigma_{\text{total}} = 0.90 \times 10^{-38} \text{ cm}^2, \quad (18)$$

a result that is approximately the same for  $\nu$  and  $\bar{\nu}$  above 2 BeV, and is essentially constant with energy. It is clear from (17) that in a measurement of the total elastic cross section, we are sensitive to the combination

$$\lambda^2 \frac{m_A^2}{3} + \frac{(2cm_E)^2}{24} \left( \frac{m_E}{m} \right)^2,$$

and not just to the axial term. If weak electricity is present, it can significantly change the total cross section, if  $m_E \approx m$  and  $c \approx 1$ . Since little experimental information is available on second-class currents, this may furnish us with a method of putting accurate limits on the weak electricity term. We shall further return to this question when we discuss time-reversal-violating nucleon polarization.

If we neglect the terms in  $F_p$  and  $F_s$  (both being proportional to  $m_{\mu}$ ) appearing in (16) and (13), we still have four independent form factors  $F_V$ ,  $F_M$ ,  $F_A$ , and  $F_E$  to be determined experimentally. Even

if second-class currents are not present in nature, we must measure at least 3 form factors,  $F_V$ ,  $F_M$ , and  $F_A$ . However, if we measure at a given  $q^2$ ,  $d\sigma/dq^2(\nu)$  and  $d\sigma/dq^2(\bar{\nu})$ , we only have two experimental numbers. One possibility is to attempt to measure these numbers as functions of energy, and thus untangle the problem. This approach may be very difficult, and an alternate suggestion, which allows an independent experimental approach, is to measure the polarization of the recoil nucleon from the reaction  $\nu + n \rightarrow p + \mu$ , in order to have a third independent experimental number at a given  $q^2$ .

To evaluate the polarization of the proton, we have chosen the laboratory reference frame shown in Fig. 1.

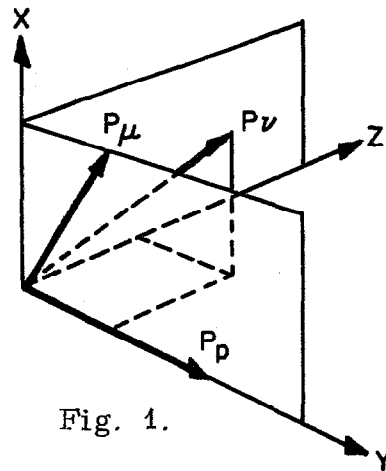


Fig. 1.

For convenience, we define a difference cross section in the direction  $(d\sigma_{\uparrow} - d\sigma_{\downarrow})/dq^2$ , where we mean by this the difference cross section between proton spins parallel and antiparallel to the  $i^{\text{th}}$  direction. Since the only components of proton polarization which can be subsequently analyzed by strong interacting scattering in the laboratory system are those perpendicular to its direction, we limit our analysis for

simplicity to the x and z directions. We neglect here the possibility of a rotation of the proton spin away from its direction due to turning in a magnetic field. In practice, if the magnetic field in a bubble chamber is large enough, we can also get information on the y direction, as pointed out by N. Christ. The extension of the results presented here for the y direction is straightforward, but tedious.

We find

$$\begin{aligned}
 \frac{(d\sigma \uparrow - d\sigma \downarrow)_x}{dq^2} = & \frac{G^2}{2\pi} \sin \theta_p \left\{ 2\lambda F_A F_V - \frac{q^2}{2m^2} \left[ (\lambda F_A)(\mu F_M) \right. \right. \\
 & - \frac{m}{E_\nu} \left\{ F_V \left( (F_V + \mu F_M - \lambda F_A) - (m_\mu b F_p) \frac{m_\mu}{2m} \right) \right. \\
 & \left. \left. - \mu F_M (m_\mu b F_p) \frac{m_\mu}{2m} \right\} \right] - \frac{q^4}{16m^4} \left( \frac{m}{E_\nu} \right) \left( (\mu F_M)^2 \right. \\
 & \left. \left. + \mu F_M (F_V - \lambda F_A) \right) \right\}. \quad (19)
 \end{aligned}$$

and

$$\begin{aligned}
 \frac{(d\sigma \uparrow - d\sigma \downarrow)_z}{dq^2} = & \frac{G^2}{2\pi} \sin \theta_p \left( \frac{\sqrt{q^2 + q^4/4m^2}}{m} \right) \left\{ 2(\lambda F_A)(c F_E) \right. \\
 & + \frac{(F_V + \mu F_M)}{2} (m_\mu a F_s) \left( \frac{m_\mu}{E_\nu} \right) + \frac{q^2}{2mE_\nu} (c F_E) \\
 & \left. \times (F_V + \mu F_M - \lambda F_A) \right\}. \quad (20)
 \end{aligned}$$



Of course, the corresponding polarization in the x and z directions are the ratios of (19) to (13) and (20) to (13), respectively. The polarizations become very simple in the high-energy limit,  $E_\nu \gtrsim 4\text{BeV}$ , and we can write them as

$$P_x = \frac{\sin \theta_p \left[ 2\lambda F_A F_V - \frac{q^2}{2m^2} (\lambda F_A)(\mu F_M) \right]}{F_V^2 + \lambda^2 F_A^2 + \frac{q^2}{4m^2} \left[ (\mu F_M)^2 + (2c F_E)^2 \right]},$$

and

$$P_z = \frac{\sin \theta_p \left( \sqrt{(q/m)^2 + q^4/4m^4} \right) (\lambda F_A)(c F_E)}{F_V^2 + \lambda^2 F_A^2 + q^2/4m^2 \left[ (\mu F_M)^2 + (2c F_E)^2 \right]}. \quad (21)$$

We note from (20) and (21) that for  $q^2 \rightarrow 0$ ,  $P_x \rightarrow \sim 1$  and  $P_z \rightarrow 0$ . In the limit of  $\lambda = 1$ , i.e. a pure V-A theory,  $P_x$  is identically 1 at  $q^2 = 0$ . In practice,  $P_x \sim 0.98$  at  $q^2 = 0$ . For small  $q^2$ , we observe that  $P_z$  increases linearly with  $q/m$ , and then levels off due to form-factor dependence on  $q^2$ . We further argue, for high  $E_\nu$ , that the kinematical factor  $\sin \theta_p$  is always about 1, for  $q^2 \ll q_{\text{max}}^2 = \frac{2mE_\nu}{1 + m/2E_\nu}$ . For neutrino energies greater than 5 BeV, for  $q^2$  up to  $2(\text{BeV}/c)^2$  the term  $\sin \theta_p$  is still within about 10% of unity. Thus, for modest  $q^2$ , we expect sizeable polarization components. Table I gives some numerical estimates of the expected polarizations, with all form factors assumed to have the same  $q^2$  dependence.

Table I.

<u>q Interval (BeV/c)</u>	<u>P<sub>x</sub> (%)</u>	<u>P<sub>z</sub> (%)</u>	<u>Percentage of Events</u>
0.0 - 0.21	~100	0	50
0.21 - 0.63	85	30	42.5
0.63 - 1.05	50		7.5

For the purpose of computing  $P_x$  in Table I, we assume  $c = 0$  (no time-reversal invariance violations). We note from Table I that the x-polarization is quite substantial, even for  $q \sim 1$  BeV/c. The z-polarization was computed using  $c = 1$  and is estimated to be  $\sim 30\%$  for  $0.2 < q < 1$  BeV/c. Thus, a measurement of the z-polarization allows us to determine the size of  $c$ , i. e., measure the strength of the second-class currents appearing in the Cabibbo scheme.

Two experimental schemes are immediately suggested to measure the nuclear polarization. In one experiment, carbon plates are placed inside the 25-ft bubble chamber filled with deuterium. Since, as we have already commented, the recoil protons are emitted close to  $90^\circ$ , the plates should be inserted parallel to the neutrino beam axis. We estimate that  $\sim 1/20$  of the protons in the momentum range 400-1000 MeV/c will give a useful scatter, with an effective polarization analysis strength of  $\sim 50\%$ . For neutrino energies above  $\sim 2$ -3 BeV, the cross section from (15) for protons with momentum above  $\sim 400$  BeV/c is about  $0.5 \times 10^{-38} \text{ cm}^2$ . Using estimates of L. Hyman<sup>1</sup> for the 25-ft bubble chamber, we expect about 5000 events per day (corresponding to this momenta interval) from the 200-BeV machine. Of these, about 2500 will have a visible proton spectator. Scaled to unit polarization

strength and unit analysis strength, this corresponds roughly to 20 useful events per day. If we assume a 30-day run, this roughly corresponds to 600 unit events, or an accuracy, for all  $q^2 \geq (400 \text{ MeV}/c)^2$ , of about 4%.

The other possible experimental arrangement, which avoids the complication of using a deuterium (as opposed to free nucleon) target, is to use the 25-ft bubble chamber filled with pure hydrogen, and use the reaction  $\bar{\nu} + p \rightarrow n + \mu^+$ , in which the neutron is detected by subsequent elastic n-p scattering, and in which the neutron polarization is measured from the angular asymmetry in n-p scattering. This scheme is most desirable because it needs no plates in the bubble chamber and is a self-contained experiment. Unfortunately, the rates are very low. The  $\bar{\nu}$  flux is down about a factor of 2 from the  $\nu$  flux. Thus, we expect about 2500 events/day. Of these, about 1/4 will have an n-p scatter, of which we estimate about only 1/2 will be visible. Thus, we shall identify only about 300/day. Of these, the analysis strength is only about 10%. Thus, scaled to unit analysis strength, the equivalent counting rate is only three per day, or about 100 per 30-day run. This number is to be contrasted with the effective 600/month for the deuterium run. However, the cleanliness of the experiment and its ease of theoretical interpretation still makes this possibility rather attractive, since at the same time, we are collecting data on  $\bar{\nu} + p \rightarrow n + \mu^+$ , and in addition the bubble chamber also samples the other  $\bar{\nu}$  reactions. For example, in

the same  $\bar{\nu}$  run, one can produce reaction (7),  $\bar{\nu} + p \rightarrow \Lambda^0 + \mu^+$ , in which the  $\Lambda^0$  analyzes its polarization automatically by its parity-violating decay. We expect (7) to be reduced by a factor of  $\sim 20 (\sin^2 \theta)$  relative to (4). Since  $2/3$  decay via the charged mode, we expect  $\sim 80$  events per day. This corresponds to  $\sim 25$  unit strength events/day. The  $\Sigma^0$  of (8) can also be used, with much lower statistical weight, since the effective  $\Lambda^0$  polarization strength from the  $\Sigma^0$  is considerably lower. Thus, strange particles can be studied in the  $\bar{\nu}$  beam, while slowly collecting information on the elastic channel.

A third class of experiments, which will not be discussed in detail here, consists of searches for new particles, such as the intermediate vector bosons  $W^\pm$ , by looking for muon pairs. One also can look for anomalous electromagnetic interactions, such as the production of muon pairs by neutrinos, e.g.,

$$\nu_\mu + Z \rightarrow \mu^+ + \mu^- + \nu_\mu + Z.$$

For these experiments, combinations of techniques using spark-chamber arrays in conjunction with large bubble chambers, both hydrogen and heavy liquid, appears most fruitful.

### Summary

Neutrino physics, via the two-body channels, appears most attractive and feasible for the study of strong-interaction form factors and currents. The experiments in which  $\nu$  and  $\bar{\nu}$  differences are compared via reactions (1) and (4), are expected to be sensitive only in the

energy region  $0.5 \leq E_\nu \leq 1.5$  BeV, and thus require a special low-energy neutrino beam.

We reiterate that for elastic scattering above about 4 BeV, the energy dependence of the cross section vanishes, which is a consequence of the locality hypothesis. This affords a particularly simple experimental test of locality.

We show that polarization furnishes us with a useful tool for disentangling the form-factor dependence of the elastic scattering cross section, and that the information gained is essentially the same for  $\nu$  and  $\bar{\nu}$  above several BeV. Thus, polarization data, in conjunction with the high-energy cross section,  $dt/dq^2$ , as well as low-energy  $\nu$  and  $\bar{\nu}$  elastic scattering differences, can, in principle, furnish us with enough information to measure the strong-interaction vector and axial vector form factors, in order to test CVC, etc.

Finally, we note that the high-energy  $\bar{\nu}$  beam is particularly rich, furnishing us with strange particle ( $\Delta S = \Delta Q$ ) form-factor information, with self-analysis of polarization, e.g. the  $\Lambda^0$  decay, as well as simultaneously giving information on the elastic channel, albeit at a rather low rate.

#### REFERENCE

- <sup>1</sup>L. Hyman, NAL Summer Study Report B.1-68-20, 1968.